

COMMENT ON THE NUMERICAL MEASUREMENTS OF THE MAGNETOHYDRODYNAMIC
TURBULENCE SPECTRUM BY A. BERESNYAK (PHYS. REV. LETT. 106 (2011) 075001; MNRAS 422 (2012)
3495; APJ 784 (2014) L20)JEAN CARLOS PEREZ¹, JOANNE MASON², STANISLAV BOLDYREV^{3,4}, AND FAUSTO CATTANEO⁵¹Space Science Center, University of New Hampshire, Durham, NH, 03824; jeanc.perez@unh.edu²College of Engineering, Mathematics and Physical Sciences, University of Exeter, EX4 4QF, UK; j.mason@exeter.ac.uk³Department of Physics, University of Wisconsin at Madison, 1150 University Ave, Madison, WI, 53706; boldyrev@wisc.edu⁴Kavli Institute for Theoretical Physics, University of California at Santa Barbara, Santa Barbara, CA, 93106⁵Department of Astronomy and Astrophysics, University of Chicago, 5640 S. Ellis Ave, Chicago, IL, 60637; cattaneo@flash.uchicago.edu(Dated: September 30, 2014)
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ABSTRACT

The inertial-interval energy spectrum of strong magnetohydrodynamic (MHD) turbulence with a uniform background magnetic field was observed numerically to be close to $k^{-3/2}$ by a number of independent groups. A dissenting opinion has been voiced by Beresnyak (2011, 2012, 2014) that the spectral scaling is close to $k^{-5/3}$. The conclusions of these papers are however incorrect as they are based on numerical simulations that are drastically unresolved, so that the discrete numerical scheme does not approximate the physical solution at the scales where the measurements are performed. These results have been rebutted in our more detailed papers (Perez et al. 2012, 2014); here, by popular demand, we present a brief and simple explanation of our major criticism of Beresnyak's work.

Subject headings: magnetic fields — magnetohydrodynamics — turbulence

The field-perpendicular inertial-interval spectrum of large-scale driven, homogeneous, strong incompressible MHD turbulence with a strong uniform magnetic field was observed numerically to be close to $-3/2$ in a number of studies (Maron & Goldreich 2001; Haugen et al. 2004; Müller & Grappin 2005; Mininni & Pouquet 2007; Chen et al. 2011; Mason et al. 2006, 2008; Perez & Boldyrev 2010; Perez et al. 2012; Chandran et al. 2014). A dissenting opinion has been voiced by Beresnyak (2011) and repeated in (Beresnyak 2012, 2014) that the spectrum is actually close to $-5/3$.

An inspection of Beresnyak's work however demonstrates that its claims are based on drastically unresolved numerical simulations, which do not represent the physical solution. We illustrate this in Fig. 1. The solid line in the top panel represents the energy spectrum of a well resolved simulation in a 512^3 mesh and Reynolds number $Re = 2400$. The dash and dash-dotted lines show the same set up with larger Reynolds numbers $Re = 6000$ and 9000 . In the latter two cases the scales at $k \gtrsim 15$ are significantly unresolved and affected by the proximity to the dealiasing cut-off $k_c = 2\pi/(3\Delta)$, where Δ is the grid size.

The proximity to the k -space cutoff is known to distort the spectral behavior at small scales in hydrodynamic simulations (Cichowlas et al. 2005; Frisch et al. 2008; Connaughton 2009; Grappin & Müller 2010); the unresolved curves in Fig. 1 bear close similarity with those results as the ratio Δ/η gets closer to 1, where η is the dissipation scale (η and Re are defined in e.g., Perez et al. 2012). Such a numerical spectral distortion has been confused by Beresnyak (2011, 2012, 2014) with the inertial interval. A standard convergence test shown in the bottom panel of Fig. 1 illustrates that the distortion is eliminated by progressively increasing the numeri-

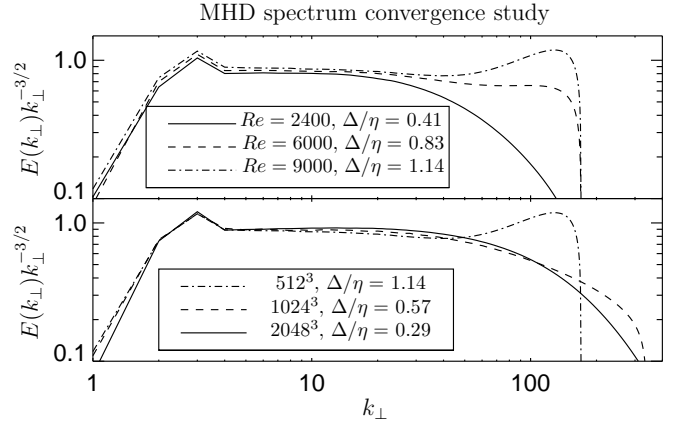


Figure 1. Convergence study of the numerical spectrum in MHD turbulence (an RMHD code is used; the numerical procedure is described in detail in (Perez et al. 2012)). Top: varying Re at fixed resolution 512^3 . The solid line represents the energy spectrum where the inertial interval is well resolved. The dash and dash-dotted lines represent similar simulations where the Reynolds number is increased so that the simulations become unresolved. As a result, at $k \gtrsim 15$ the numerical solution does not approximate the physical one: the numerical spectrum steepens and then flattens closer to the cut-off scale, which is a purely numerical effect. Bottom: varying resolution at fixed $Re = 9000$. The dash-dotted line shows the spectrum of the most unresolved simulation where an unphysical distortion is present at small scales ($k \gtrsim 15$). This numerical distortion progressively disappears as the resolution is increased without changing the physical parameters of the simulations (the dashed-dotted and solid curves); the inertial interval now extends to about $k \sim 30$.

cal resolution to 1024^3 and 2048^3 keeping all the physical parameters unchanged.

In the resolved runs, the energy spectrum agrees with the $-3/2$ scaling; see the solid lines in Fig. 1. However, in the unresolved runs, the scaling of the unresolved part of the spectrum is steeper and closer to $-5/3$, see Fig. 2.

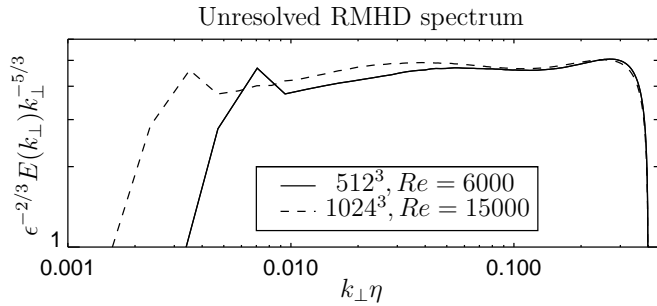


Figure 2. The scaling of the unresolved numerical spectrum in two runs with different resolutions and Reynolds numbers. The Re numbers are chosen such that $\Delta/\eta \approx 0.83$, and this ratio is the same in both runs. Both simulations are essentially unresolved at small scales, $k\eta \gtrsim 0.1$, where the solution of the numerical scheme scales differently from the inertial interval.

This is not surprising: the scaling of the discrete numerical scheme does *not* have to agree with the scaling of the physical solution if the former does not approximate the latter (e.g., Perez et al. 2012, 2014). The scaling of the unresolved part in Fig. 2 was incorrectly attributed by Beresnyak (2011, 2012, 2014) to the scaling of the physical solution, as the standard procedure (described in Fig. 1) to test the numerical convergence was *not* followed in these works.

Due to the presence of the Alfvénic velocity scale, the Kolmogorov first self-similarity hypothesis cannot be formulated in MHD turbulence, and the energy spectrum cannot be established based solely on dimensional arguments (more discussion can be found in Perez et al. (2014)). In this case, numerical studies should be conducted with the utmost care. The incorrect conclusions of (Beresnyak 2011, 2012, 2014) could be avoided if their numerical simulations followed the standard procedures (as in Fig. 1), where first and foremost, the numerical convergence is checked before any meaningful measurements are made.

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